Solving Fuzzy Multi-objective Optimization Using Non-dominated Sorting Genetic Algorithm II

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***Abstract*—This paper presents the stages for solving fuzzy multi-objective optimization problems using genetic algorithm approach. Before applying non-dominated sorting genetic algorithm II (NSGA II) techniques to obtain optimal solution, first multi-objective possibilistic (fuzzy) programming was converted into an equivalent auxiliary crisp model to form deterministic programming model. To determine the best solution from Pareto set, we implied feasibility degree of decision variable and satisfaction degree of decision maker. The best optimal solution is the intersection between α-feasibility degree and satisfaction degree of the decision makers that has the highest fuzzy membership degree. For numerical experiment, we used simple formulation in multi-objective fuzzy linear programming model with three maximum objective functions, three decision variables, and six constraints. The comparison of the results shows that our results are better for two objectives than that of compromising programming.**

Keywords*— *fuzzy numbers, NSGA II, satisfaction degree, multi-objective optimization

I. Introduction

In the real world, decision maker often faces several objectives that must be satisfied simultaneously and several objectives sometimes conflict occur. To satisfy the objectives are required multi-objective optimization approach for solving the problem. Currently, there are some techniques to be performed for solving the multi-objective optimization (MOO) problems. Generally, MOO techniques classified into two classes, which are classical and evolutionary method [1]. The classical method converts the multi-objective problem into single-objective by aggregating objective functions which optimize the most important objective and perform the other objectives as constraints. Evolutionary methods are imitating natural evolution principle that generate stochastic searching and optimization algorithm. Non-dominated sorting genetic algorithm II(NSGA II) is one of evolutionary algorithm model that presents computation and identification of Pareto front set efficiently through surviving diversity without using various additional parameters [1].

Moreover, we often encounter the problem of uncertainty, imprecision, and vagueness such as product demand, product quality, lead time, production rate, etc. Zadeh [2] introduced fuzzy numbers set to describe the ambiguity, vagueness, and uncertainty in an assessment or measurement. In fuzzy set theory, each fuzzy set is defined by the membership function where the value is in the interval 0 and 1. Fuzzy numbers in the form of a mathematical equation are called as fuzzy mathematical programming. Inuiguchi and Ramik [3] classified fuzzy mathematical programming into two main classes, namely: 1) flexible programming, which is fuzzy programming of ambiguity when there is flexibility on the target value given in objective function and constraints flexibility, and 2) possibilistic programming, which is fuzzy programming of coefficients vagueness in the objective function and constraints.

Several optimization methods have been developed to solve fuzzy multi-objective model. Li and Yang [4] converted fuzzy multi-objective formulation into single-objective model and then optimized by using genetic algorithm approach. Fuzzy multi-objective optimization (fuzzy programming) model can be transformed into deterministic multi-objective optimization before employing genetic algorithm approach to solve the problem [5]. Xu et al. [5] used expected value operator based on [6] and chain-constrained operator to convert fuzzy programming into deterministic programming. Mahnam et al. [7] developed an inventory model for assembling supply chain network with demand and reliability of suppliers in fuzzy condition. They used Particle Swarm Optimization (PSO) approach to obtain optimum solution. Yazdiana et al. [8] established fuzzy multi-objective mixed integer linear programing model and then performed numerical solution using compromise programming method. Ghasemy Yaghin et al. [9] transformed the fuzzy non- linear multi-objective model into an equivalent multi-objective crisp model. They converted multi-objective model into single-objective model using fuzzy goal programming method. To obtain the optimum solution they conduct the optimization using particle swarm optimization (PSO) approach. Arenas Parra et al. [10] solved fuzzy multi-objective problem using compromise programming approach. Torabi and Hassini [11] used novel interactive fuzzy method (called TH method) approach to solve multi-objective linear programming problem. Peidro et al. [12] converted fuzzy multi-objective linear programming (FMOLP) model into auxiliary crisp single-objective linear using TH fuzzy programming method. Özceylan and Paksoy [13] employed the weighted average method to convert fuzzy constraint model into crisp constraint model. The initial model of fuzzy multi-objective optimization is converted into equivalent ordinary linear programming using auxiliary variable L. Completing literature review about multi-objective optimization can be seen in [14].

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Several methods and procedure aforementioned have been developed to solve fuzzy MOO problems. In general, we can classify the scopes and the procedures solving fuzzy MOO problems involving: 1) the mathematic formulation models, 2) the converting method from fuzzy programming to deterministic programming, and 3) the using of optimization method both classical and evolutionary methods.

This paper presents framework for solving fuzzy multi-objective using NSGA II approach. We present fuzzy (possibilistic) programming in multi-objective optimization model. NSGA II is employed to obtain optimal solutions set (Pareto front set). Before applying NSGA II technique, first, fuzzy programming was transformed into deterministic programming by converting the fuzzy parameters into crisp based on the fuzzy ranking method based [15]. After the deterministic model was formed then the optimization is conducted using NSGA II technique at the different value of α-feasibility degrees. According to [16] the higher α feasibility degree is the worse the solution to be obtained. To solve this problem, feasibility degree and satisfaction degree of decision maker for each objective to be achieved should be balanced.

The main contribution of this paper is to solve fuzzy multi-objective using NSGA II through incorporating several previous works. We incorporate the fuzzy ranking method, NSGA II [1], fuzzy decision making [17], and the final solution selection from Pareto front set developed from Jiménez approach [16] in which they employed it to solve fuzzy linear programming.

For result evaluation, we compare our result with previous work using compromise programming in [10].

II. CONVERTING OF FUZZY PROGRAMMING FORMULATION INTO DETERMINISTIC PROGRAMMING

Fuzzy programming is transformed into deterministic programming by changing the fuzzy numbers into crisp model based on Jiménez [15] approach. Fuzzy ranking method converts possibilistic programming into deterministic programming based on concept of mathematical expected interval (EI) and the expected value of fuzzy numbers (EV). Expected interval (EI) and the expected value of fuzzy numbers (EV) for triangular fuzzy numbers can be defined in equation 1 and fuzzy distribution is illustrated in Fig. 1.

$$\begin{aligned} EI(\tilde{A}) &= [E_1^A, E_2^A] \\ &= [\int_0^1 f_A^1(x)dx, \int_0^1 g_A^1(x)dx] = [\frac{1}{2}(a^p + a^m), \frac{1}{2}(a^m + a^o)] \\ EV(\tilde{A}) &= \frac{E_1^A + E_2^A}{2} = \frac{a^p + 2a^m + a^o}{4} \end{aligned} \quad (1)$$

Where E_1^A and E_2^A are the lower and upper limit of fuzzy number interval A (EI($\tilde{A}$)), respectively. In the same way, the expected interval (EI) and the expected value (EV) for trapezoidal fuzzy numbers $\tilde{A}$ can be determined in equation 2 and illustrated in Fig. 2.

$$\begin{aligned} EI(\tilde{A}) &= [E_1^A, E_2^A] \\ &= [\int_0^1 f_A^1(x)dx, \int_0^1 g_A^1(x)dx] = [\frac{1}{2}(a^p + a^{m_1}), \frac{1}{2}(a^{m_2} + a^o)] \\ EV(\tilde{A}) &= \frac{E_1^A + E_2^A}{2} = \frac{a^p + a^{m_1} + a^{m_2} + a^o}{4} \end{aligned} \quad (2)$$

All pairs of fuzzy numbers $\tilde{A}$ and $\tilde{B}$, in which the degree of $\tilde{A}$ is greter than that of $\tilde{B}$ based on [16] and [18] are given as follows:

$$\mu_M(\tilde{A},\tilde{B}) = \begin{cases} 0 & \text{jika } E_2^A - E_1^B < 0 \\ \dfrac{E_2^A - E_1^B}{E_2^A - E_1^B - (E_1^A - E_2^B)} & \text{jika } 0 \in (E_1^A - E_2^B, E_2^A - E_1^B) \\ 1 & \text{jika } E_1^A - E_2^B > 0 \end{cases} \quad (3)$$

Where $[E_1^A, E_2^A]$ and $[E_1^B, E_2^B]$ are expected interval (EI) A and B respectively. If $\mu_M(\tilde{A},\tilde{B}) \geq \alpha$ then it will be stated that $\tilde{A}$ is greater or equal to $\tilde{B}$ at least in degree α represented as $\tilde{A} \geq_\alpha \tilde{B}$. If $\mu_M(\tilde{A},\tilde{B}) = 0.5$ then the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are equal. For all pairs of fuzzy numbers $\tilde{A}$ and $\tilde{B}$, it is stated that $\tilde{A}$ is equal to $\tilde{B}$ in degree α, if there is relation then the equation is given below:

$$\tilde{A} \geq_{\alpha/2} \tilde{B}, \quad \tilde{A} \leq_{\alpha/2} \tilde{B} \quad (4)$$

Equation (4) can be rewritten as follows:

$$\frac{\alpha}{2} \leq \mu_M(\tilde{A},\tilde{B}) \leq 1 - \frac{\alpha}{2} \quad (5)$$

According to [16], decision vector $x \in \Re^n$ is feasible in degree α if $\min_{i=1,\dots,m}\{\mu_M(\tilde{A},\tilde{B})\} = \alpha$. If fuzzy mathematical programming model with all parameters are triangular or

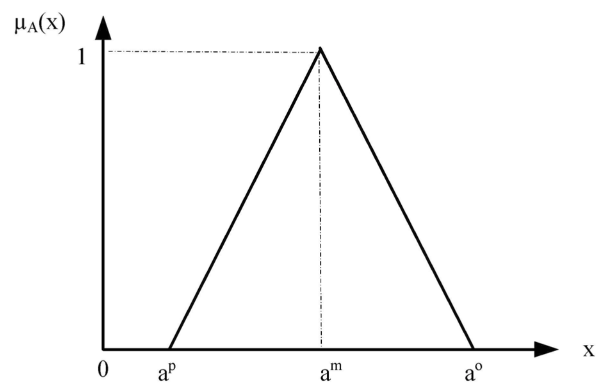

Fig. 1. Distribution of triangular fuzzy for parameter $\tilde{A}_i$

trapezoidal fuzzy numbers then the equation $\tilde{A}_i x = \tilde{B}_i$, equivalent to the following equation:

$$\frac{\alpha}{2} \leq \frac{E_2^{A_i x} - E_1^{B_i}}{E_2^{A_i x} - E_1^{A_i x} + E_2^{B_i} - E_1^{B_i}} \leq 1 - \frac{\alpha}{2}, \quad i = 1+1, \dots, m \quad (6)$$

The given equation of $\tilde{A}_i x \geq \tilde{B}_i$ is equivalent to:

$$\frac{E_2^{A_i x} - E_1^{B_i}}{E_2^{A_i x} - E_1^{A_i x} + E_2^{B_i} - E_1^{B_i}} \ge \alpha, \quad i = 1, ..., l \qquad (7)$$

Equation 6 can be rewritten to form equivalent crisp α-parametric model as follows:

$$[(1-\frac{\alpha}{2})E_2^{A_i} + \frac{\alpha}{2}E_1^{A_i}]x \ge \frac{\alpha}{2}E_2^{B_i} + (1-\frac{\alpha}{2})E_1^{B_i}, \quad i = l+1, ..., m \qquad (8)$$

$$[\frac{\alpha}{2}E_2^{A_i} + (1-\frac{\alpha}{2})E_1^{A_i}]x \le (1-\frac{\alpha}{2})E_2^{B_i} + \frac{\alpha}{2}E_1^{B_i}, \quad i = l+1, ..., m \qquad (9)$$

Equation (7) can be rewritten to form equivalent crisp α-parametric model as follows:

$$[(1-\alpha)E_2^{A_i} + \alpha E_1^{A_i}]x \ge \alpha E_2^{B_i} + (1-\alpha)E_1^{B_i}, \quad i = 1, ..., l \qquad (10)$$

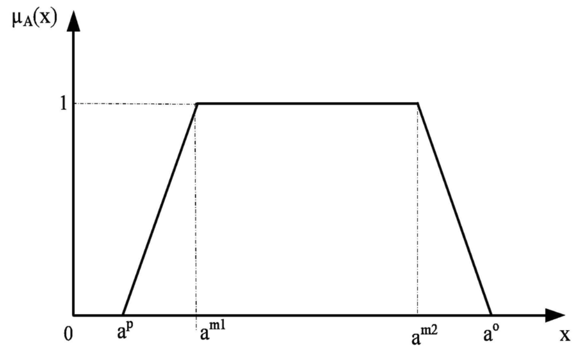

Fig. 2. Distribution of trapezoidal fuzzy for parameter $\tilde{A}$

If constraint is in the form of $\tilde{A}_i x \le \tilde{b}_i$, the crisp equivalent equation can be written as follows:

$$[(1-\alpha)E_1^{A_i} + \alpha E_2^{A_i}]x \le \alpha E_1^{B_i} + (1-\alpha)E_2^{B_i}, \quad i = 1, ..., l \qquad (11)$$

Where α is determined by the decision maker, which is the degree of the least fulfilled constraint, or α is the minimum acceptable feasibility degree of decision vector in which the value is between 0 and 1. The higher α value, the higher feasibility degree of decision variable.

III. Decision Making of Fuzzy Multi-objective Optimization

After possibilistic programming was converted into deterministic programming then multi-objective optimization using NSGA II can be conducted to obtain Pareto fronts set based on the value of α-feasibility degree. The results of Pareto front set usually prove that the lower of α-feasibility degrees, the greater constraint model to be violated. The decision makers will not take high risks by violating the constraint. To make the best decision of the results of multi-objective optimization on the different value of α feasibility degree then the approach introduced by Jiménez et al. (2007) is performed. Besides using α-feasibility degree Jiménez approach also employs satisfaction degree of decision makers to obtain fuzzy membership degree for each solution of Pareto front.

The decision makers are asked to state their satisfaction level of very satisfied and not satisfied about constraint of the results of the objective function. For the maximization objective function, the decision makers will be more satisfied if the result of the objective function is bigger, and vice versa. For minimization objective function, the decision makers will feel satisfied if the result of the objective function is smaller. In this case, the fuzzy membership function of the very satisfied decision-makers is 1, and that of very dissatisfied decision-makers is 0 in which it can be formulated in (12) and (13).

For maximization objective function:

$$\mu(f_i(\alpha)) = \begin{cases} 0, & \text{if } f_i(\alpha) \le f_i(\alpha)^{min} \\ \frac{f_i(\alpha) - f_i(\alpha)^{min}}{f_i(\alpha)^{max} - f_i(\alpha)^{min}}, & \text{if } f_i(\alpha)^{min} \le f_i(\alpha) \le f_i(\alpha)^{max} \\ 1, & \text{if } f_i(\alpha) \ge f_i(\alpha)^{max} \end{cases} \qquad (12)$$

For minimization objective function:

$$\mu(f_i(\alpha)) = \begin{cases} 1, & \text{if } f_i(\alpha) \le f_i(\alpha)^{min} \\ \frac{f_i(\alpha)^{max} - f_i(\alpha)}{f_i(\alpha)^{max} - f_i(\alpha)^{min}}, & \text{if } f_i(\alpha)^{min} \le f_i(\alpha) \le f_i(\alpha)^{max} \\ 0, & \text{if } f_i(\alpha) \ge f_i(\alpha)^{max} \end{cases} \qquad (13)$$

Where $\mu(f_i(\alpha))$ is the fuzzy membership function i on α-feasibility degree, $f_i(\alpha)$ is the value the objective function i. $f_i(\alpha)^{min}$ and $f_i(\alpha)^{max}$ are the value of the minimum and maximum limit of the objective function i, respectively.

After satisfaction degree of each objective function on α-feasibility degree was obtained then fuzzy decision membership degree for each objective function is calculated using the following equation.

$$\mu_{\tilde{D}}(f_i(\alpha_j)) = \alpha_j . \mu(f_i(\alpha_j)) \qquad (14)$$

Where $\mu_{\tilde{D}}(f_i(\alpha_j))$ is the fuzzy membership objective function i for the j^{th} α-feasibility degree. $\mu(f_i(\alpha_j))$ is the satisfaction degree of decision makers for objective i for j^{th} α-feasibility degree.

The fuzzy final decision for optimal solution by determining membership degree for each solution is intersection of all membership degrees of objective function i at the j^{th} solution. Determining the degree of membership intersection based on [17] can be written as (14).

$$\mu\left(f_i\left(\alpha_j\right)_D\right) = \alpha_j.\mu(f_1\left(\alpha_j\right)) \cap \alpha_j.\mu(f_2\left(\alpha_j\right)) \cap .... \cap \alpha_j.\mu(f_i\left(\alpha_j\right))$$
$$= \text{Min}(\alpha_j.f_1\left(\alpha_j\right),\ \alpha_j.f_2\left(\alpha_j\right), ...,\ \alpha_j.f_i\left(\alpha_j\right)) \qquad (15)$$

The final decision for the optimal solution is those having the maximum value of $\mu\left(f_i\left(\alpha_j\right)_D\right)$ or can be written as (16).

$$\textbf{Max}\ \mu\left(f_i\left(\alpha_j\right)_D\right) = \textbf{Max}\left(\textbf{Min}(\alpha_j.f_1\left(\alpha_j\right),\ \alpha_j.f_2\left(\alpha_j\right), ...,\ \alpha_j.f_i\left(\alpha_j\right))\right) \qquad (16)$$

IV. Methods

This study aims to solve fuzzy multi-objective problem using evolutionary approach. The steps required to solve the problems of fuzzy multi-objective optimization in this study include:

1. Determining mathematic formulation

   For numerical experiment, we solved simple formulation which is fuzzy multi-objective linear programming model

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or called by possibilistic programming with fuzzy parameters in objective functions and constraints.

2. Converting fuzzy programming formulation into deterministic programming
   This step aims to simplify employing NSGA II to solve fuzzy MOO problems and to yield objective function values in crisp numbers.
   Fuzzy linear programming model is converted into deterministic linear programming by changing the parameters of fuzzy numbers into crisp numbers based on fuzzy ranking method introduced by [15] and its application to solve linear programming by [16].
3. Optimizing using genetic algorithm approach
   The multi-objective optimization is performed using non-dominated sorting genetic algorithm II (NSGA II) developed by [1]. Recently, NSGA II is most employed to solve MOO problems. Deb [1] developed NGSA II to tackle some drawbacks of NSGA [19] involving high complexity computation for identification of non-dominated sorting, inability surviving diversity, and Lack of specification of sharing parameter.
   The numeric experiment is conducted NSGA II using different α-feasibility degree which are between 0-1. The solution of fuzzy multi-objective approach optimization employs the approach of NSGA II using tool aids of MOEA Framework version 2.8 available in http://www.moeaframework.org/ and coded in Java Netbeans.
4. Determining the final decision
   After Pareto front was obtained based on the feasibility degree then the decision making is conducted to select the best final solution. The results of Pareto front set usually prove that the lower α-feasibility degrees, the greater model constraint is violated. To make the best decision of the optimization results of the objective functions on the different α-feasibility degree then the approach introduced by Jiménez et al. method [16] was conducted.

V. Numerical Experiment

A. Formulation

For numerical experiment, we used simple formulation example in form multi-objective fuzzy linear programming in [10].

$$\text{Max } F1 = (40, 50, 80)x_1 + 100x_2 + 17.5x_3 \quad (17)$$

$$\text{Max } F2 = (80, 90, 120)x_1 + (50, 75, 110)x_2 + 50x_3 \quad (18)$$

$$\text{Max } F3 = (10, 25, 70)x_1 + 100x_2 + 75x_3 \quad (19)$$

s.t.

$$(6, 12, 14)x_1 + 17x_2 \leq 1400 \quad (20)$$

$$3x_1 + 9x_2 + (3, 8, 10)x_3 \leq 1000 \quad (21)$$

$$10x_1 + (7, 13, 15)x_2 + 15x_3 \leq 1750 \quad (22)$$

$$(4, 6, 8)x_1 + 16x_2 \leq 1325 \quad (23)$$

$$(7, 12, 19)x_2 + 16x_3 \leq 900 \quad (24)$$

$$9.5x_1 + (3.5, 9.5, 11.5)x_2 + 4x_3 = (1060, 1075, 1080) \quad (25)$$

Equation (17) to (19) are objective functions in maximization and equation (20) to (21) are constraint functions in equality and inequality. Coefficients or parameters in parenthesis are fuzzy numbers which are defined as triangular fuzzy numbers.

Before fuzzy multi-objective linear programming model above solved by NSGA II, first it transformed into deterministic programming using equation (1), objective functions can converted into deterministic programming as follows:

$$\text{Max } F1 = \frac{40 + 2(50) + 80}{4} x_1 + 100x_2 + 17.5x_3$$

$$\text{Max } F2 = \frac{80 + 2(90) + 120}{4} x_1 + \frac{50 + 2.70 + 110}{4} x_2 + 50x_3$$

$$\text{Max } F3 = \frac{10 + 2(25) + 70}{4} x_1 + 100x_2 + 75x_3$$

Using equation (7) to (10), fuzzy parameters in the left and right-hand constraints converted into crisp numbers that form the equivalent auxiliary crisp model or deterministic programming, as follows:

$$\left[(1-\alpha)\left(\frac{6+12}{2}\right) + \alpha\left(\frac{12+14}{2}\right)\right]x_1 + 17x_2 \leq 1400$$

$$3x_1 + 9x_2 + \left[(1-\alpha)\left(\frac{3+8}{2}\right) + \alpha\left(\frac{8+10}{2}\right)\right]x_3 \leq 1000$$

$$10x_1 + \left[(1-\alpha)\left(\frac{7+13}{2}\right) + \alpha\left(\frac{13+15}{2}\right)\right]x_2 + 15x_3 \leq 1750$$

$$\left[(1-\alpha)\left(\frac{4+6}{2}\right) + \alpha\left(\frac{6+8}{2}\right)\right]x_1 + 16x_2 \leq 1325$$

$$\left[(1-\alpha)\left(\frac{7+12}{2}\right) + \alpha\left(\frac{12+19}{2}\right)\right]x_2 + 16x_3 \leq 900$$

$$9.5x_1 + \left[\left(1-\frac{\alpha}{2}\right)\left(\frac{9.5+11.5}{2}\right) + \frac{\alpha}{2}\left(\frac{3.5+9.5}{2}\right)\right]x_2 + 4x_3 \geq \frac{\alpha}{2}\left(\frac{1075+1080}{2}\right) + \left(1-\frac{\alpha}{2}\right)\left(\frac{1060+1075}{2}\right)$$

$$9.5x_1 + \left[\frac{\alpha}{2}\left(\frac{9.5+11.5}{2}\right) + \left(1-\frac{\alpha}{2}\right)\left(\frac{3.5+9.5}{2}\right)\right]x_2 + 4x_3 \leq \left(1-\frac{\alpha}{2}\right)\left(\frac{1075+1080}{2}\right) + \left(\frac{\alpha}{2}\right)\left(\frac{1060+1075}{2}\right)$$

B. Applying NSGA II to solve the fuzzy multi-objective optimization problems

Multi-objective optimization is performed to obtain the optimum solution set or called Pareto front set. In this study, NSGA II technique was employed using α value from 0.1 to 1 that was coded in Java Netbeans according to MOEA Framework version 2.8 which available in http://www.moeaframework.org/. NSGA II parameters was used including: the population size 200, the number of genes 3, the number of generation 10,000, employing Simulated Binary Crossover (SBX) algorithm with SBX parameter rate of 0.9 and distribution index of 20, and using polynomial mutation operator with mutation rate 0.1 and distribution index of 20.

The results of the numerical calculations of fuzzy multi-objective optimization using NSGA II method α value between 0.1 and 1 can be seen in Table 1.

TABLE 1. THE EVALUATION OF FUZZY MULTI-OBJECTIVE PROBLEMS WITH DIFFERENT α-FEASIBILITY DEGREE

α	F_1	F_2	F_3	x_1	x_2	x_3
0.1	9058.52	10483.61	11047.79	33.54	63.79	47.72
0.2	8530.13	11316.41	8541.12	70.83	41.46	27.91
0.3	8310.01	11256.06	8204.32	73.56	37.92	26.95
0.4	8871.29	10018.14	11007.71	27.34	65.30	47.85
0.5	7410.72	11155.76	7728.74	78.39	24.66	36.20
0.6	7403.13	11100.98	7700.75	77.85	24.98	35.64
0.7	8271.71	10508.27	9631.75	48.92	48.32	42.80
0.8	7149.80	11005.72	7406.16	79.73	21.40	35.66
0.9	7278.59	10988.75	9683.47	57.61	29.84	64.37
1	8059.43	9726.88	9677.00	37.25	52.63	42.71

C. *Determining the final decision*

The lower α-feasibility degree value, the greater model constraint is violated. The decision makers will not take high risk by breaking the constraint. However, the greater α-feasibility degree value gives the better results. To make the best decision multi-objective optimization result on the different value of α-feasibility degree then the approach introduced by [16] is performed. Besides using α-feasibility degree, Jiménez approach also employs satisfaction degree of decision makers to obtain fuzzy membership degree for each solution of Pareto front.

Suppose that the decision maker is very satisfied if the results of objective 1 are more than 9058.52 and he/she does not want less than 7149.80. According equation (12), satisfaction degree of decision maker for the objective 1 can be stated in equation (26). Furthermore, in the same way, satisfaction degree of decision maker for the objective 2 and 3 can be determined that stated in equation (27) and (28), respectively.

$$\mu(f_1(\alpha_j)) = \begin{cases} 1, & \text{if } f_1(\alpha_j) \geq 9058.52 \\ \dfrac{f_1(\alpha_j) - 7149.80}{9058.52 - 7149.80}, & \text{if } 7149.80 \leq f_1(\alpha_j) \leq 9049.80 \\ 0, & \text{if } f_1(\alpha_j) \leq 7149.80 \end{cases} \tag{26}$$

$$\mu(f_2(\alpha_j)) = \begin{cases} 1, & \text{if } f_2(\alpha_j) \geq 11316.41 \\ \dfrac{f_2(\alpha_j) - 9726.88}{11316.41 - 9726.88}, & \text{if } 9726.88 \leq f_2(\alpha_j) \leq 11316.41 \\ 0, & \text{if } f_2(\alpha_j) \leq 9726.88 \end{cases} \tag{27}$$

$$\mu(f_3(\alpha_j)) = \begin{cases} 1, & \text{if } f_3(\alpha_j) \geq 11047.79 \\ \dfrac{f_3(\alpha_j) - 7406.16}{11047.79 - 7406.16}, & \text{if } 7406.16 \leq f_3(\alpha_j) \leq 11047.79 \\ 0, & \text{if } f_3(\alpha_j) \leq 7406.16 \end{cases} \tag{28}$$

Using equation (26) to (28) then satisfaction degree for each objective can be calculated on different α values that can be seen in Table 2.

For calculation example:

In Table 1, let $f_1(0.1)$= 9058.52 (for objective 1 (F_1) at α=0.1) then according to equation (26) that is obtained $\mu(f_1(0.1))=1$.

For $f_1(0.2)$= 8530.13 then obtained:

$\mu(f_1(0.2))=(8530.13-7149.80)/(9058.52-7149.80)=0.723$

After satisfaction degree for each objective function at α-feasibility degree was obtained then the fuzzy decision membership degree is calculated for each objective function using equation 14.

Suppose that the satisfaction degree of objective 1 at α = 0.1 and $\mu(f_1(\alpha_{0.1}))$ = 1 then the fuzzy decision membership degree $\mu_{\tilde{D}}(f_1(\alpha_{0.1}))$ =0.1(1) =0.1. In the same way, fuzzy decision membership degree for each objective at satisfaction degree α can be determined as shown in Table 2.

TABLE 2. SATISFACTION DEGREE AND FUZZY DECISION MEMBERSHIP DEGREE AT α-FEASIBILITY DEGREE FOR OBJECTIVE I at α-FEASIBILITY DEGREE

α	$\mu(f_1(\alpha))$	$\mu(f_2(\alpha))$	$\mu(f_3(\alpha))$	$\alpha_j.\mu(f_1(\alpha))$	$\alpha_j.\mu(f_2(\alpha))$	$\alpha_j.\mu(f_3(\alpha))$	min $\alpha_j.\mu(fi(\alpha j))$
0.1	1.000	0.476	1.000	0.100	0.048	0.100	0.048
0.2	0.723	1.000	0.312	0.145	0.200	0.062	0.062
0.3	0.608	0.962	0.219	0.182	0.289	0.066	0.066
0.4	0.902	0.183	0.989	0.361	0.073	0.396	0.073
0.5	0.137	0.899	0.089	0.068	0.449	0.044	0.044
0.6	0.133	0.864	0.081	0.080	0.519	0.049	0.049
0.7	0.588	0.492	0.611	0.411	0.344	0.428	0.344
0.8	0.000	0.805	0.000	0.000	0.644	0.000	0.000
0.9	0.067	0.794	0.625	0.061	0.714	0.563	0.061
1	0.477	0.000	0.624	0.477	0.000	0.624	0.000

In Table 2, we can see that the best solution is at 0.7-feasibility degree because it has the highest final decision membership degree, which is 0.344. If the decision makers are not satisfied with the final solution, the decision makers can alter the fuzzy boundary of objectives or select α-feasibility degree. In Table 1 can be seen that the best solution is at 0.7-feasibility degree in which the value of decision variable x_1,

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x_2, and x_3, are 48.92, 48.32, 42.80, respectively. Those decision variable values result 8271.71, 10508.27, 9631.75 for the value of objective function 1, 2, and 3, respectively.

TABLE 3. COMPARISON OF THE RESULTS OF NSGA II AND COMPROMISE PROGRAMMING FOR SOLVING FUZZY MOO PROBLEMS

	Compromise programming results for β=0.8 [10]		NSGA II results (at 0.7- feasibility degree)
	L_1	$L\infty$	
Objective 1	[5420.14, 7232.07]	[5402.38, 7249.83]	8271.71
Objective 2	[10204.34, 12207.79]	[10210.65, 12231.79]	10508.27
Objective 3	[5244.91, 7962,80]	[5052.63, 7823.80]	9631.75
x_1	90.6	92.37	48.92
x_2	6.38	5.79	48.32
x_3	40.28	38.01	42.8

For evaluation, we compare our results with the previous work using compromise programming in [10]. We can see in Table 3 that the results of comparison between NSGA II approach and compromise programming. The results of compromise programming are in expected intervals of fuzzy numbers that need further stages to obtain exact numbers. NSGA II approach yields optimal solution in crisp numbers and more optimal (maximum) solution at objective 1 and 2 than that of compromise programming.

VI. Conclusions

This paper presents fuzzy multi-objective linear programming or also called as possibilistic programming model solving by NSGA II. First, possibilistic programming formulation is converted into deterministic programming by changing the parameters of fuzzy numbers into crisp. After deterministic programming was formed then NSGA II technique is applied to obtain a set of Pareto front with α-feasibility degrees between 0.1 to 1. Pareto front set obtained is a collection of optimal decision for the issues discussed. The final optimal solution is the intersection between α-feasibility degree and satisfaction degree of the decision makers that generates fuzzy membership degree. The final optimal fuzzy solution is the one having the highest fuzzy membership degree. The comparison of optimization results shows that NSGA II approach yields more maximum for two objectives than that of compromising programming.

In this paper, we imply simple formulation for numerical experiment. For next study, this framework can be implied to solve fuzzy multi-objective optimization problems for complex formulations and models.

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