A Model to Minimize Disagreement in Group Decisions with Consensus

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Abstract

The group decision making (GDM) is a procedure where a chance is given for team members to impact a decision. Therefore, team members feel obligated to the decision that is made, such a decision-making procedure needs some degree of consensus. Decision making problems mainly involve discovering the best option from a feasible alternative set. We devise an integer program model for facing the difference between everyone's preferences and the final solution. The consensus degree can be discovered to reveal how far a group of individuals is from maximum consensus. The model is created in order to minimize the disagreement of the consensus obtained.

Keywords: *Decision making, Group decision, Disagreement, Integer programming.*

1.   Introduction

Regularly, in an organization an unstructured decision problem turns up and requires to be resolved, which means that the authorized should determine concerning to solving the problem. The process implicated is called decision making which can be characterized as a set of activities whose goal is to find suitable solutions for the problem. A finite set of substitutes must be defined and evaluated. From this analysis, decision makers, judges or expert(s) must determine which of the options is the best one to be selected to solve an issue in [1].

Although the best choice has been decided and suggested it still required to be processed by a team member, called group decision making (GDM). The GDM is a procedure whereby an opportunity is given for team members to impact a decision. Consequently, team members feel obligated to the decision that is made [2]; such a decision-making process needs some degree of consensus. In a very practical case study stated by Wilkof (1989) a group of production engineers explain consensus as the point where there are no additional protests, or the absence of sabotage or interference in the selected activity. Additional understanding into the definition of consensus shows that it is a process that is dependent upon the open and honest discussion of everyone's recommendations [4]. Other techniques of GDM consist conflict and debate. For instance, devil's advocacy or dialectical inquiry where the ideas and assumptions of participants are introduced and then are systematically assessed and challenged [5] and [6]. One can also look at other options to reaching a decision by consensus, that is majority rules or the voting method. While commonly used, getting a decision with these procedures often guides to a decline in commitment in the application of the decision [7]. In fact, a GDM procedure involves discussion and ultimately a solution that leaves no disagreement among participants.

Group decision making has been extensively studied since group decision making procedures are very usual in many fields. Formal representation of the experts' views, aggregation of evaluations or choice of the best options have been some of major areas referred by scientists and researchers.

In GDM problems there are two procedures to perform before achieving a final solution (Harrera et al., 2004; [8]) the consensus procedure and [9] the selection procedure. The first process refers to how to gain the maximum degree of consensus or agreement between the set of experts on the solution set of options, while the second process based on how to achieve the solution set of options from the opinions on the choices given by the experts. Obviously, it is desirable that the set of experts reach a high degree of consensus on the solution set of options.

Consensus has become a main area of research in GDM ([10], [8], Palomares et al., 2014; [9], [11] and [12]) Normally, at the beginning of every GDM problem, experts' opinions may vary significantly. Thus, it is required to devise a consensus procedure to achieve a solution of consensus. Classically, consensus is described as the full and unanimous agreement of all the experts concerning all the possible choices. This definition is problematic for our objectives for two reasons. 1) First, it only lets us to distinguish between two states, that is, the existence and absence of consensus. 2) Second, the opportunities for reaching such a full agreement are rather low. Moreover, complete agreement is not essential in real life.

In this paper, we focus on another promising area, the study of group decision making processes from the concept of impact and social networks. In order to do so, we offer a novel model that collects the experts' initial opinions and gives an outline to characterize the impact of a given expert over the other(s). With this proposal it is feasible to guess both the evolution of the group decision making process and the final solution before implementation the group discussion process and therefore predicting possible actions.

A social network can be identified as a set of people or groups of people which has some pattern of interactions or ties between them [13] and [14]. These patterns could be friendship among a group of individuals, in industry there are business relationships, and for families we have intermarriages. These are all instances of networks that have been studied in the past. From these examples we can say that social network impacts can be utilized for identifying human behaviour. People interact with different numbers of individuals and with some individuals more than others and this influences behaviour in fundamental ways.

Normally, a network is utilized to gain information on social interactions. Everyone is signified by a node in the network, and there is an edge between two nodes if a social interaction has happened at any point in time between the two individuals signified by these nodes. The conceptualization of social systems as graphs and networks suggested the chance for systematic investigation and theorizing of the structure of relations among social actors beyond the pair. Whereas classical sociology tended to make a quantum leap from the individual and the pair to the triple, group, or society, graph theory presented the tools to formally illustrate and visualize social structure involving three and more actors.

Let $N = \{1, 2,\ldots, n\}$ be a set of network nodes, with each node signifying a social actor. The actors are often persons, but may also be groups, organizations or other social entities.

A graph can be utilized to signify social network in a way of specifying relationships between each node of a network. The correlation is signified by links called edges.

Utilizing graph, this network model of social interactions has an obvious perception mathematically. Sadly, from the structure point of view, this model has a main disadvantage is that it is broadly static in that all information about the dynamic relationship between actors is removed. The static nature of the model can give imprecise or incorrect information about patterns in the social activities of actors.

In optimization point of view, we can utilize the concept of centrality to illustrate the measurement whether an actor's position is the most significant (or popular). The concept of centrality as utilized to social communication was established already by Bavelas (1950), since then many diverse measures of centrality have been suggested (see, for example, [16], Borgatti, 1995, [17], [13], [18] [19] and [20]).

2.   Social Network Model

We develop a model for social network based on graph formulation defined in prior Section.

The purpose of the social network model is to maximize the centrality. The constraints of the model consist of Density of a network's connectivity (D), Betweenness centrality (B), and Closeness centrality (Cl).

The model can be formulated as a 0-1 integer programming problem.

$$\max \sum_{i\in\delta^{-1}(i);(i,j)\in E;i,j\notin x} C_j x_{ij} \tag{1}$$

Subject to

$$\sum_{(i,j)\in\delta^+(i)} x_{ij} \leq D_i \qquad \forall i, j \in N; i, j \notin X \tag{2}$$

$$\sum_{(i,j)\in\delta^-(i)} x_{ij} = \sum_{(v,j)\in\delta^+(i)} x_{ij} \qquad i \notin X; \forall i \in N \tag{3}$$

$$\sum_{(i,j)\in\delta^+(i)} \left(\tau_{ji} x_{ij}\right) \geq B_i \qquad i, j \notin X; \forall i \in E \tag{4}$$

$$\sum_{(i,j)\in\delta^-(i)} \left(\tau_{ij} x_{ij}\right) \leq Cl_i \qquad i, j \notin X; \forall i \in E \tag{5}$$

$$x_{ij} \in \{0,1\} \qquad (i, j) \notin X, \forall (i, j) \in E \tag{6}$$

τ_{ij} Consumption or prevalence factor.

3.   Modelling Social Network Dynamic

There are numerous crucial points are essentially to be fulfilled in order we can say that a person (actor) has a dynamic interaction in the social network. These points are:

a)   The number of outdegree ties,

b)   Reciprocal relationship,

c)   Transitivity interaction, and

d)   Equilibrium.

Now we can devise the model with the goal to maximize degree of centrality, the number of outdegree ties, and reciprocity relationship. The model can be devised as a binary integer programming problem, which can be written mathematically as follows.

$$\max \sum_{i\in\delta^{-1}(i);(i,j)\in E;i,j\notin x} C_j x_{ij} + \sum_{(i,j)\in E} \delta_i^+ x_{ij} + \sum_{(i,j)\in E} \rho x_{ij} \tag{7}$$

Subject to

$$\sum_{(i,j)\in\delta^+(i)} x_{ij} \leq D_i \qquad \forall i, j \in N; i, j \notin X \tag{8}$$

$$\sum_{(i,j)\in\delta^-(i)} x_{ij} = \sum_{(v,j)\in\delta^+(i)} x_{ij} \qquad\qquad i\notin X; \forall i\in N \qquad (9)$$

$$\sum_{(i,j)\in\delta^+(i)} \left(\tau_{ji} x_{ij}\right) \geq B_i \qquad\qquad i,j\notin X; \forall i\in E \qquad (10)$$

$$\sum_{(i,j)\in\delta^-(i)} \left(\tau_{ij} x_{ij}\right) \leq Cl_i \qquad\qquad i,j\notin X; \forall i\in E \qquad (11)$$

$$x_{ij} \in \{0,1\} \qquad\qquad (i,j)\notin X, \forall (i,j)\in E \qquad (12)$$

Consensus measurement can be stated as follows.

$$CL(V^1, V^2, \ldots, V^n) = \frac{1}{nml}\sum_{k=l}^{n} d(V^k, V^C)) \qquad (13)$$

where $d(V^k, V^C)$ is Manhattan distance between V^k and V^C

$$d(V^k, V^C) = \sum_{i=1}^{m}\sum_{j=1}^{l} \left|v_{ij}^k - v_{ij}^c\right|, k = 1, 2, \ldots, n \qquad (14)$$

If $CL(V^1, V^2, \ldots, V^n) = 0$, then all experts should have full and unanimous consensus with the collective opinion.

Normally, the optimization model of consensus rule based on distance can be devised as follows.

$$
\begin{cases}
min\dfrac{1}{ml}\displaystyle\sum_{k=1}^{n} d\left(V^k, \overline{V^k}\right) \\[2ex]
s.t \begin{cases}
min\dfrac{1}{nml}\displaystyle\sum_{k=1}^{n} d\left(V^k, \overline{V^k}\right) \leq \varepsilon \\[2ex]
\overline{v_{ij}^c} = F_w^{OWA}\left(\overline{v_{ij}^1}, \overline{v_{ij}^2}, \ldots, \overline{v_{ij}^n}\right) \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, l
\end{cases}
\end{cases} \qquad (15)
$$

Where $\overline{V^k}, (k = 1, 2, \ldots, n)$ and $\overline{V^c}$ are the decision variables.

The consensus optimization model related to social network can be stated as in the Eq. (15), in which the expression of Eq. (11) can be written as

$$\sum_{(i,j)\in\delta^-(i)} \tau_{ij} x_{ij} \leq CL(V^1, V^2, \ldots, V^N), \qquad i,j\notin X; \quad \forall i\in E \quad (16)$$

Given that the value of $\overline{V^k}$ can be acquired from the optimal result of linear program Eq. (15).

4.   The Algorithm

To solve the 0-1 integer programming model, we approve the method of examining a decreased problem in which most of the integer variables are held constant and only a small subset permitted varying in discrete steps.

The steps of the procedure can be summarized as follows.

Step 1.   Solve the problem ignoring integral needs.

Step 2.   Gain a (sub-optimal) integer-feasible solution, utilizing heuristic rounding of the continuous solution.

Step 3.   Split the set I of integer variables into the set I_1, at their bounds that were non-basic at the continuous solution, and the set I_2, $I = I_1 + I_2$.

Step 4.   Do a search on the objective function, preserving the variables in I_1 non-basic and allowing only discrete adjustments in the values of the variables in I_2.

Step 5.   At the solution in step 4, analyse the decreased costs of the variables in I_1. If any should be liberated from their bounds, add them to the set I_2 and repeat from step 4, otherwise terminate.

It should be stated that the above procedure gives an outline for the advancement of specific strategies for classes of problems.
The integer results are maintained in super basic variables set. Then we conduct an integer line search to enhance the integer feasible solution [21].

5.   Conclusions

This paper presents a model to explore group decision making processes from the concept of impact and social networks. We address a novel model that collects the experts' initial opinions and gives an outline to characterize the impact of a given expert over the other(s). With this model it is feasible to gain both the evolution of the group decision making process and the final solution before applying to the group discussion process and such a way to predict possible actions. The model created based on graph theory is in the form of integer programming. We propose a direct search method for solving the model.

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